On the Non-Lorentz-Invariance of M.W. Evans' O(3)-Symmetry Law

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Abstract

In 1992 Evans proposed the $O(3)$ symmetry of electromagnetic fields by adding a constant longitudinal ghost magnetic field to the well-known transverse electric and magnetic fields of circularly polarized plane waves, and has since then elevated this so-called symmetry to the status of a *new law of electromagnetics*. As a law of physics must be invariant under admissible coordinate transforms, namely Lorentz transforms, in 2000 he published a proof of the Lorentz invariance of $O(3)$ symmetry of electromagnetic fields. This proof is incorrect, because it is erroneously premised on the wave number and frequency being invariant under Lorentz transforms. The simple removal of this erroneous premise is sufficient to show that the $O(3)$ symmetry is not Lorentz invariant, and thus is not a valid law of physics. Furthermore, as the $O(3)$ symmetry later became the basic assumption of Evans' covariant grand unified field theory (CGUFT), recently renamed as the Einstein-Cartan-Evans (ECE) theory), this theory is also physically invalid.

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1 M.W. Evans' $O(3)$ hypothesis

The assertion of $O(3)$ symmetry is a central concern of M.W. Evans' research since 1992: He claims that the transverse magnetic field of circularly polarized electromagnetic plane wave is accompanied by a constant longitudinal field $\mathbf{B^{(3)}}$, the so-called "ghost field".

Evans considers a circularly polarized plane electromagnetic wave propagating along the z axis of a Cartesian coordinate system [1, Chap.1.2] . Using the electromagnetic phase[1]

[1, (38)]
$$\Phi = \omega t - \kappa z,$$

where t denotes time, $\kappa = \omega/c$ is the free-space wave number, ω is the angular frequency, and c is the speed of light in free space, Evans describes the wave in terms of his complex circular basis [1, (1.41)]. The total magnetic field is stated by him as

[1, (43/1)]
$$\mathbf{B}^{(1)} = \tfrac{1}{\sqrt{2}}B^{(0)}(\mathbf{i} - i\mathbf{j})e^{i\Phi},$$

[1, (43/2)]
$$\mathbf{B}^{(2)} = \tfrac{1}{\sqrt{2}}B^{(0)}(\mathbf{i} + i\mathbf{j})e^{-i\Phi},$$

[1, (43/3)]
$$\mathbf{B}^{(3)} = B^{(0)}\mathbf{k},$$

where $i = \sqrt{-1}$ and $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ are the Cartesian unit vectors. The total magnetic field satisfies his "cyclic $O(3)$ symmetry relations"

[1, (44/1)]
$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*},$$

[1, (44/2)]
$$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)}\mathbf{B}^{(1)*},$$

[1, (44/3)]
$$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)}\mathbf{B}^{(2)*}.$$

Equation [1, (43/3)] specifically *defines* Evans' ghost field $\mathbf{B}^{(3)}$, which is coupled by the relations [1, (44)] to the transverse components stated as [1, (43/1)] and [1, (43/2)].

Evans' **B Cyclic Theorem** is the statement that the magnetic field compo-

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[1]Equations from Evans' book on the so-called covariant grand unified field theory (CGUFT) [1] appear with equation labels [1, (nn)] in the left margin.

nents [1, (43/1)] and [1, (43/2)] of each circularly polarized plane wave are accompanied by a longitudinal component [1, (43/3)], and that all three components together fulfil the cyclic $O(3)$ symmetry relations [1, (44)]. Evans considers this $O(3)$ hypothesis as a **Law of Physics**.

2 Is the $O(3)$ hypothesis Lorentz-invariant?

A law of physics must be invariant under admissible coordinate transforms, namely Lorentz transforms. A circularly polarized plane wave displays this property when described by any observer who is moving with uniform velocity with respect to an inertial frame K at rest. Therefore, EVANS' $O(3)$ symmetry law should be valid in all inertial frames of reference. Hence, to check the physical validity of Evans' $O(3)$ hypothesis, we shall apply a longitudinal Lorentz transform to the plane wave as described by Evans (the ghost field included).

In an article published in 2000 [2, p.14], Evans provided a proof of the Lorentz invariance of the $O(3)$ hypothesis [1, (44)] by referring to the invariance of the vector potential $\mathbf{A}$ under Lorentz transforms. That is a good method obtaining the transformed magnetic field, *if done correctly*.

The vector potentials of the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$ of the plane wave under consideration are given by

$$\left.\begin{array}{l} \mathbf{A}^{(1)} = \frac{1}{\kappa}\mathbf{B}^{(1)} = \frac{1}{\kappa\sqrt{2}}B^{(0)}(\mathbf{i} - i\mathbf{j})e^{i\Phi} \\[2mm] \mathbf{A}^{(2)} = \frac{1}{\kappa}\mathbf{B}^{(2)} = \frac{1}{\kappa\sqrt{2}}B^{(0)}(\mathbf{i} + i\mathbf{j})e^{-i\Phi} \end{array}\right\}, \tag{1}$$

while the vector potential of the *longitudinal* component $\mathbf{B}^{(3)}$ is

$$\mathbf{A}^{(3)} = \frac{1}{2}\mathbf{B}^{(3)} \times (x\mathbf{i} + y\mathbf{j}) = \frac{1}{2}B^{(0)}(x\mathbf{j} - y\mathbf{i}). \tag{2}$$

The Lorentz invariance of the vector potential $\mathbf{A}^{(3)}$ yields the Lorentz invariance of $\mathbf{B}^{(3)}$ and the factor $B^{(0)}$ in [1, (43)] and [1, (44)] for *longitudinal* Lorentz transforms (i.e. between inertial frames K and K' such that K' moves relative to K with velocity $\mathbf{v} \parallel \mathbf{k}$ and $\beta = |\mathbf{v}|/c$).

Evans [2] ignored that ω and κ are *not* Lorentz-invariant. Under *longitudinal* Lorentz transforms, we have the well-known Doppler effect:

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$$\omega' = \sqrt{\frac{1-\beta}{1+\beta}}\,\omega, \qquad\qquad \kappa' = \sqrt{\frac{1-\beta}{1+\beta}}\,\kappa. \qquad (3)$$

Therefore the invariance of the vector potentials *does not transfer* to the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$.

More importantly here, due to the invariance of $\mathbf{A}$ we obtain

$$\mathbf{B}'^{(1)} \times \mathbf{B}'^{(2)} = \kappa'^2 \mathbf{A}'^{(1)} \times \mathbf{A}'^{(2)} = \frac{1-\beta}{1+\beta}\,\kappa^2 \mathbf{A}^{(1)} \times \mathbf{A}^{(2)} = \frac{1-\beta}{1+\beta}\,\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}, \quad (4)$$

from (1). That is, the expression $\mathbf{B}^{(1)} \times \mathbf{B}^{(2)}$ on the left side of [1, (44/1)] is not Lorentz-invariant, whereas $\mathbf{B}^{(3)}$ on the right side of [1, (44/1)] is Lorentz-invariant due to (2). Equation [1, (44/1)] therefore, if valid in the inertial frame K, *cannot be valid* also in the inertial frame K'. Hence, Evans' cyclic $O(3)$ symmetry relations [1, (44)] are *not Lorentz-invariant and so cannot be a Law of Physics.*

3   Epilog

After the foregoing sections had appeared on http://arxiv.org, Evans reacted polemically on
http://www.atomicprecision.com/blog/2006/12/26/jackson-on-the-lorentz-transformation/.
Therein, he suggested that Equation (3.111) of another book of his [4] should prove the Lorentz-invariance of his "B Cyclics".

This equation is as follows:

[4, (3.111)] $$\qquad\qquad\qquad B^{(0)\prime} = [\tfrac{1-\frac{v}{c}}{1+\frac{v}{c}}]^{1/2} B^{(0)}.$$

It is a consequence of the Lorentz transform applied to the transverse components $\mathbf{B}^{(1)}$ and $\mathbf{B}^{(2)}$. However, just before [4, (3.111)]) Evans stated the transformation rule

[4, (3.110-3)] $$\qquad\qquad\qquad \mathbf{B}^{(3)\prime} = \mathbf{B}^{(3)}, \dots$$

This rule implies that $|\mathbf{B}^{(3)\prime}| = |\mathbf{B}^{(3)}|$ which immediately leads to a *contradiction* in Ref. [4] itself, as on p.5 of that book we find the equation

[4, (1.2a)]
$$|\mathbf{B}^{(1)}| = |\mathbf{B}^{(2)}| = |\mathbf{B}^{(3)}| = B^{(0)}$$

that yields

$$B^{(0)\prime} = |\mathbf{B}^{(3)\prime}| = |\mathbf{B}^{(3)}| = B^{(0)}$$

in contrast to [4, (3.111)]. Therefore, Evans stopped his consideration in Ref. [4] just before recognizing the final contradiction he would have arrived at.

References

[1] M.W. Evans, Generally Covariant Unified Field Theory, the geometrization of physics; Arima 2006

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[4] M.W. Evans, The Enigmatic Photon, vol. 4, Kluwer Academic Publishers 1997